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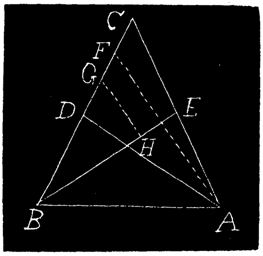
III. Solution by the PROPOSER.

Let the bisectors BE and AD be equal, prove triangle ABC isosceles.

Three suppositions are possible.

1st, $\angle A > \angle B$; 2nd, $\angle A < \angle B$; 3rd, $\angle A = \angle B$.



First, suppose $\angle A > \angle B$: then $\frac{1}{2}\angle A > \frac{1}{2}\angle B$. Construct $\angle FAD = \angle CBE$. Then in the triangle FAB, $FB > FA$ (greater side opposite greater angle). Lay off on BF a distance BG equal to AF, and draw GH parallel to FA. Then the triangle BGH=triangle FAD ($BG = FA$, by construction, $\angle GBH = \angle FAD$, for the same reason, $\angle BGH = \angle DFA$, exterior-interior angles.)

$\therefore$ $DH = BA$ (homologous sides of equal triangles) *which is absurd*, because $BE = AD$, by hypothesis, and BH is only a part of BE.

Second, in a similar manner it can be shown that $\angle B$ cannot be greater than A; *i. e.* $\angle A$ cannot be less than $\angle B$.

Third, as $\angle A$ can neither be greater nor less than $\angle B$, it must be equal to $\angle B$. $\therefore$ the triangle is isosceles. Q. E. D.

For other demonstrations of this problem, see Vol. II., pages 158, 189—192. EDITOR.

86. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Prove that the four conics which have S for focus and which touch the three sides of each of the triangles ABC, AEF, BFD, CDE, have their latera-recta equal.

Solution by the PROPOSER.

Reciprocate with respect to S; then we have the three altitudes of an equilateral triangle passing through a point, and the circumscribing circles of the four triangles formed by joining the feet of the perpendiculars equal.

The latera recta of the given conics are then equal.

87. Proposed by WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military Academy, Washington, Miss.

Given any two straight lines in space, AB, CD, which do not intersect. So construct upon one of the lines as base, a triangle, having its vertex in the other line, such that its perimeter shall be a minimum.

No solution of this problem has been received.

88. Proposed by FREDERICK R. HONEY, Ph. B., Instructor in Mathematics in Trinity College, New Haven, Conn.

Prove that the volume of the frustum of a cone is equal to one-sixth of the altitude multiplied by the sum of the areas of the upper base, the lower base, and four times the area of the section midway between the upper and lower bases.

Solution by FREMONT CRANE, Sand Coulee, Mont.; ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa; G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va., and the PROPOSER.

Let R=radius of the lower base; r=radius of the upper base; ρ=radius of the base mid way between the upper and lower bases ; and a=altitude of frustum. Then $\rho=\frac{1}{2}(R+r)$. $\therefore 4\rho^2=(R+r)^2$.

Volume of frustum$=\frac{1}{3}\pi a(R^2+r^2+Rr)=\frac{1}{6}\pi a(2R^2+2r^2+2Rr)=\frac{1}{6}\pi a[R^2+r^2+(R+r)^2]=\frac{1}{6}\pi a(R^2+r^2+4\rho^2)$.

The same method applies to frustums of pyramids, and all solids coming under the prismatoid formula as special cases.

Also solved in a more general manner by *B. F. SINE*, *P. S. BERG*, *J. SCHEFFER*, *HARVEY N. DAVIS*, *W. H. DRANE*, and *CHAS. C. CROSS*.

CALCULUS.

68. Proposed by **EDWARD DRAKE ROE, JR., A. M., Associate Professor of Mathematics, Oberlin College, Oberlin, Ohio.**

If $a^{x^{\cdot^{\cdot^{x}}}}$ to r steps be denoted by $a^{\overset{r}{x}}$, and if $y=x^{\overset{r}{x}}$, prove that

$$D_xy=x^{\overset{r}{x}+\overset{r-1}{x}+\dots\overset{1}{x}}(\log x)^{r-1}(1+\log x)+\sum_{k=2}^{k=r}x^{\overset{r}{x}+\overset{r-1}{x}+\dots\overset{k-1}{x}-1}(\log x)^{r-k}.$$

Solution by the PROPOSER.

If $y=f_1(x)^{f_2(x)}$, we obtain, by taking the logarithm of both sides of the equation, and differentiating, the formula

$$D_xy=f_1(x)^{f_2(x)}\log f_1(x)D_xf_2(x)+f_1(x)^{f_2(x)-1}f_2(x)D_xf_1(x).$$

If in this $f_1(x)=x$, $f_2(x)=x^x$, that is if $y=x^{\overset{x}{x}}=\overset{2}{x}$, we obtain

$$D_xy=x^{\overset{2}{x}+\overset{1}{x}}\log x(1+\log x)+x^{\overset{2}{x}+\overset{1}{x}-1},$$

and the formula to be proved is true when $r=2$. It is evidently not true for values of $r<2$. Assume that it is true for all other values of r

Let $y=x^{\overset{r+1}{x}}=x^{\overset{r}{x}}$. In the above formula put, $f_1(x)=x$, $f_2(x)=\overset{r}{x^x}$, and we get

$$D_xy=x^{\overset{r+1}{x}}\log xD_x\overset{r}{x^x}+x^{\overset{r+1}{x}-1}\overset{r}{x^x},$$

but by this assumption this is

$$D_xy=x^{\overset{r+1}{x}}\log x[x^{\overset{r}{x}+\overset{r-1}{x}+\dots\overset{1}{x}}(\log x)^{r-1}(1+\log x)]+x^{\overset{r+1}{x}}\log x\sum_{k=2}^{k=r}x^{\overset{r}{x}+\overset{r-1}{x}+\dots\overset{k-1}{x}-1}(\log x)^{r-k}$$
$$+x^{\overset{r+1}{x}+\overset{r}{x}-1}$$
$$=x^{\overset{r+1}{x}+\overset{r}{x}+\overset{r-1}{x}+\dots\overset{1}{x}}(\log x)^r(1+\log x)+\sum_{k=2}^{k=r+1}x^{\overset{r+1}{x}+\overset{r}{x}+\dots\overset{k-1}{x}-1}(\log x)^{r+1-k}.$$

But this expression has the same form with respect to $r+1$, that the assumption had with respect to r, and since the assumption was true for $r=2$, it is also true for all values of r greater than 2, which is what we had to prove.

Erlangen, Bayern, Hauptstrasse 83II, 26 February, 1898.

Also solved by *C. W. M. BLACK*, *W. W. LANDIS*, and *G. B. M. ZERR*.